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# Seesaw induced Higgs mechanism

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**Abstract.** We discuss a two scalar doublets model which induces the Higgs mechanism by means of a seesaw mechanism. This model naturally predicts a light Higgs scalar whose mass is suppressed by the grand unification scale. The model requires an intermediate scale between the electroweak symmetry breaking scale and the grand unification scale at $10^9$ GeV. Below this intermediate energy scale the usual standard model appears as an effective theory. An implementation of this mechanism in models where the Planck scale is in the TeV region is discussed.

The electroweak symmetry breaking in the standard model [1], which is a gauge theory based on the structure group $SU(3) \times SU(2) \times U(1)$, is implemented by means of the Higgs mechanism [2]. If the standard model is embedded into a grand unified theory like e.g. $SU(5)$ [3] or $SO(10)$ [4], it suffers from two problems that have often been discussed in the literature. The running of the coupling constant of the standard model suggests that the unification is taking place at a grand unification scale $\Lambda_{\rm GUT} \approx 10^{16}\,$GeV. The gauge hierarchy problem states that it is unnatural for the electroweak breaking scale $\Lambda_{\rm EW} \approx 174\,$GeV to be so small compared to the fundamental scale $\Lambda_{\rm GUT}$.

A second potential problem with the standard model is that the Higgs boson is an elementary scalar field. It is useful, in order to get some intuition, to regularize the theory using a cutoff $\Lambda$. One finds that the Higgs squared boson mass $m_H^2$ receives quadratic "corrections"

$$m_H^2 \approx {m_H^0}^2 + \frac{3g^2\Lambda^2}{32\pi^2 m_W^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4\sum_f \frac{n_f}{3} m_f^2\right), \tag{1}$$

where $g$ is the $SU(2)$ gauge coupling, $m_W$ and $m_Z$ are the masses of the electroweak gauge bosons, $m_f$ stands for a fermion mass and where the sum runs over the fermion flavors. The standard model is a renormalizable theory [5]. However, if the intuitive cutoff $\Lambda$ which is used to regularize the theory, is identified with the grand unification scale $\Lambda_{\rm GUT}$, it seems to require an unnatural adjustment to keep the Higgs boson mass small compared to the scale $\Lambda_{\rm GUT}$. This is the naturalness problem [6].

Besides these problems, the standard model has another unpleasant feature: the sign of the Higgs boson doublet squared mass has to be chosen to be negative to trigger the Higgs mechanism. In other words the Higgs boson doublet is a tachyon. This may be a signal that some mechanism triggering the phase transition is missing. We nevertheless want to stress that the Higgs mechanism as it is usually formulated is completely consistent. The terms of a potential can be positive or negative, but it would be more satisfactory to have a mechanism that fixes the signs of the parameters.

Different scenarios have been proposed to cure these problems. A dynamical effect could be at the origin of this phase transition; see [7] for a review. In that case it is not necessary to introduce elementary bosons in the model. A supersymmetric extension of the standard model [8] is also conceivable. But in that case the problem of breaking the gauge symmetry is shifted to that of supersymmetry breaking which remains unsolved. Other scenarios that are solving the hierarchy problem by lowering the Planck scale and possibly also the scale for grand unification to the TeV region have been proposed [9].

In this paper, we present a minimal extension of the standard model which can be seen as a limit case of a two Higgs doublets model [10]. We consider the same action as that of the standard model but with a modified scalar sector. The first scalar boson is denoted by $h$ and the second scalar doublet by $H$. Both doublets have exactly the same quantum numbers as the usual standard model Higgs doublet. The Yukawa sector involves only the boson $h$. The scalar potential is chosen according to

$$\begin{aligned} S_{\rm scalar} = &- \int d^4x \left(h^\dagger H^\dagger\right) \begin{pmatrix} \epsilon^2 & m^2 \\ m^2 & M^2 \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} \\ &- \int d^4x \lambda_h (h^\dagger h)^2 - \int d^4x \lambda_H (H^\dagger H)^2 \\ &- \int d^4x \lambda_3 (H^\dagger H)(h^\dagger h) - \int d^4x \lambda_4 (H^\dagger H)(H^\dagger h) \end{aligned} \tag{2}$$

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$$-\int \mathrm{d}^4x \lambda_5 (H^\dagger h)(h^\dagger h).$$

At this stage the electroweak gauge symmetry is still unbroken.

The mass squared $\epsilon^2$ of the first scalar boson $h$, is assumed to be zero as in the Coleman–Weinberg case [11]. Taking radiative corrections into account, one expects that the boson $h$ will get a small, possibly negative, squared mass according to the Coleman–Weinberg mechanism [11]. An early analysis of this mechanism yielded a mass for the first scalar boson of the order of 10 GeV [12]. In the Coleman–Weinberg case, the scalar boson mass is a calculable quantity and turns out to be small, one could thus argue that in that particular case, i.e. when the tree level mass of the scalar boson is vanishing, one obtains a light scalar boson mass when radiative corrections are taken into account. Nevertheless, this calculation involves a renormalization procedure and thus does not solve the naturalness problem as it is usually stated.

Furthermore, we also assume that the coefficients of the terms of the type $\bar{h}H\bar{H}H$, $\bar{H}h\bar{h}h$ and $\bar{h}h\bar{H}H$ are very small after renormalization and can safely be neglected. The model we are considering is renormalizable. In a renormalizable theory these coefficients are not calculable and can have any arbitrary values.

There is nevertheless a case of special interest. If $h$ and $H$ are composite objects, the couplings of the effective theory are driven to special values by renormalization equations, i.e. by fixed point conditions. A possibility is to drive the couplings $\lambda_3$, $\lambda_4$ and $\lambda_5$ in the infrared region. This can be achieved in theories with condensates and these terms will be small enough to be neglected. One nevertheless has to assume the condition $\epsilon^2 \ll m^2 \ll M^2$.

We want to stress that the Coleman–Weinberg condition and the intermediate scale are introduced to show that a small Higgs boson mass, relative to other scales of the problem, can be obtained from a seesaw condition. As we shall see, the mass obtained by the Coleman–Weinberg mechanism for the first boson $h$ is assumed to be small compared to the two other scales $m$ and $M$ involved in the model, and this contribution can be neglected. On the other hand, we shall assume that the mass of the second boson $H$ is large and typically of the order of the fundamental scale of the model. The usual argument is that the mass of the second boson $H$ receives a large contribution because its "naked" mass is non-vanishing, and if the model is embedded into a grand unified theory, a possible mass scale for the mass of the boson $H$ is the grand unification scale. One thus expects $M \sim \Lambda_{\mathrm{GUT}}$. In that case we have a large hierarchy and it is difficult to understand why the scale of the electroweak interactions is so small compared to $\Lambda_{\mathrm{GUT}}$.

On the other side, the situation is quite similar to the situation in neutrino physics where a large Majorana scale is used to explain a small neutrino mass; see [13] for reviews. Such a situation also appears in flavor physics where the light quark masses are solely induced by mixing with heavy quarks [14]. If the parameter $m$ is not too large and not too small, a seesaw mechanism [15] can be applied. We shall discuss values of $m$ for which this mechanism can be applied later. The seesaw mechanism has been applied in top–color condensation models [16] to generate the electroweak symmetry breaking and it has also recently been applied to a supersymmetric model in an attempt to solve the so-called $\mu$ problem [17].

After diagonalization of the mass matrix in (2) using

$$R = \begin{pmatrix} 1 & \frac{m^2}{M^2} \\ -\frac{m^2}{M^2} & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3}$$

we obtain the squared masses of the mass eigenstates

$$\mathcal{M}^2 \approx \begin{pmatrix} -\frac{m^4}{M^2} & 0 \\ 0 & M^2 \end{pmatrix}. \tag{4}$$

The first boson $h$ has become a Higgs boson with a negative squared mass given by

$$m_h^2 = -\frac{m^4}{M^2}, \tag{5}$$

whereas the second scalar boson $H$ has a positive squared mass of the order of $\Lambda^2_{\mathrm{GUT}}$ and is thus not contributing to the electroweak symmetry breaking. The mass of the physical Higgs boson is given by

$$M_h^{\mathrm{phys}} = \sqrt{2}\frac{m^2}{M}. \tag{6}$$

One finds that a Higgs boson mass of the order of 100 GeV can be obtained if $m \sim 10^9$ GeV using $\Lambda_{\mathrm{GUT}} \sim 10^{16}$ GeV. The mass scale $m \sim 10^9$ GeV is quite natural in e.g. grand unified models broken with an intermediate breaking scale [18]. The parameter $m^2$ also receives quadratic "corrections" due to loop diagrams involving the electroweak bosons. A natural scale for the parameter $m^2$ is

$$\Lambda_m^2 \sim \Lambda_{\mathrm{CW}} \Lambda_{\mathrm{GUT}}, \tag{7}$$

where $\Lambda_{\mathrm{CW}} \approx 80$ GeV is fixed by the mass scale of the gauge bosons giving rise to the Coleman–Weinberg mass [11]. The intermediate scale $\Lambda_m$ is a natural one because the loop diagrams connect the heavy scalar boson $H$, whose typical scale is $\Lambda_{\mathrm{GUT}}$, to the light scalar boson $h$, whose typical scale is the mass scale generated by the Coleman–Weinberg mechanism. This is an intuitive argument which is in the spirit of the seesaw mechanism when applied to neutrinos. In the neutrino case [15,13], the off-diagonal terms, i.e. the Dirac masses, are assumed to be naturally of the order of the electroweak scale because the Dirac neutrino is a $SU(2)$ doublet. On the other hand the Majorana neutrino's typical mass is of the order of the grand unification because it is a $SU(2)$ singlet. Our case is analogous; the difference is that we apply the seesaw mechanism to the squared mass matrix, and the off-diagonal terms are not necessarily related to any physical quantities, but are determined in order to connect the two scales. One finds $\Lambda_m \sim 9 \times 10^8\,\text{GeV} \sim m$. This intermediate mass scale also appears in $SO(10)$ grand unification models with an intermediate breaking scale [18] where it roughly corresponds to the scale of the breaking of the $SU(4)_C$ Pati–Salam gauge group [19]. Inserting $m \sim 9 \times 10^8\,\text{GeV}$ into (6) yields $M_h^{\text{phys}} = \mathcal{O}(100\,\text{GeV})$, which should not be taken as a calculation of the Higgs boson's mass but rather as a confirmation that a Higgs boson mass in the 100 GeV region is natural.

Another way to stabilize the intermediate scale $\Lambda_m$ would be to require that the off-diagonals terms, e.g. $m^2 h^\dagger H$, arise from an interaction of the type $N^\dagger N h^\dagger H$, $N$ being a new scalar field, which yields a term $v_N^2 h^\dagger H$, where $v_N^2$ is the vacuum expectation value of the operator $N^\dagger N$. Note that the boson $N$ does not need to be charged under $SU(2)$. The scale of the mass of the boson $N$ could be stabilized by another seesaw–Higgs mechanism with a new intermediate scale between $10^9\,\text{GeV}$ and $\Lambda_{\text{GUT}}$. This new scale might again be stabilized by a new intermediate scale and we can introduce as much scalar doublets as necessary to insure the stabilization of these mass scales and so on till the grand unification scale is reached.

As in the standard model we can make use of the unitarity gauge to rotate the first doublet $h$:

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \eta + v \end{pmatrix}, \tag{8}$$

where the vacuum expectation value of Higgs boson is given by

$$v = \sqrt{\frac{m^4}{M^2 \lambda_h}} = \frac{m^2}{M} \frac{1}{\sqrt{\lambda_h}}, \tag{9}$$

which is naturally a small number. The electroweak scale $\Lambda_{\text{EW}}$ can be defined by $\Lambda_{\text{EW}} = v/\sqrt{2}$ and is thus a small number too.

The masses of the electroweak bosons are given as usually by $m_W = gv/2$ and $m_Z = m_W \sqrt{1 + g'^2/g^2}$, where $g'$ is the $U(1)$ gauge coupling. The fermion masses are generated by the Yukawa mechanism as we had assumed a Yukawa type coupling between the doublet $h$ and the fermions. The four heavy degrees of freedom contained in $H$ decouple from the remaining of the model and we are left at energies well below the intermediate scale $m$ with the usual standard model. It should be noted that one could relax the assumption concerning the absence of Yukawa couplings between the heavy scalar doublet $H$ and the standard model fermions as flavor changing neutral current transitions are suppressed by the grand unification scale; note also that the heavy scalar doublet $H$ is not developing a vacuum expectation value. If the doublet $H$ has Yukawa type couplings, it leads to point-like four-fermions interactions which are suppressed by the grand unification scale, once the heavy degrees of freedom have been integrated out of the theory. It is conceivable, if the Yukawa couplings are strong enough, that fermion condensates will form [20]. The electroweak symmetry breaking could be a combination of two effects: a fundamental scalar boson and a composite scalar boson could both contribute to the gauge symmetry breaking, in the spirit of the model proposed in [21].

In our framework we can turn to our advantage the fact that fundamental bosons receive large quadratic corrections or are nearly massless. The Higgs mechanism appears as a consequence of the seesaw mechanism, and is triggered by the large gauge hierarchy. We note that the dual description of the standard model presented in [22], allowing a calculation of the weak mixing angle and of the Higgs boson mass, remains valid in this framework because the hierarchy between the two mass scales is huge.

In models where the grand unification takes place at a very high scale of the order of $10^{16}\,\text{GeV}$, our model will be difficult to distinguish from the standard model, whereas it might be easier to do so in models where the unification scale is lower [9]. The diagonalized squared mass matrix $\mathcal{M}^2$ is actually defined via an expansion in the parameter $M^2 x$ with $x = \frac{m}{M}$, given by

$$\mathcal{M}^2 \approx \begin{pmatrix} -\frac{m^4}{M^2} + M^2 x^8 - \ldots & 0 \\ 0 & M^2 + M^2 x^4 - \ldots \end{pmatrix} \tag{10}$$

The expansion parameter of the rotation matrix (3) is only of the order $x$ and the corrections are thus always very small as long as $\Lambda_{\text{GUT}} \gg m$. If we take $\Lambda_{\text{GUT}} = 1000\,\text{GeV}$ and $m = 266\,\text{GeV}$, one finds that the corrections to the Higgs boson squared mass are small and that it is still negative. Our model in that framework provides a natural mechanism to trigger the Higgs mechanism. The four heavy degrees of freedom would have masses in the TeV range. If the grand unification scale is in the TeV region and if the second boson $H$ has Yukawa type couplings with the standard model fermions, we expect that flavor changing neutral current transitions should take place at an observable rate.

We should like to conclude by emphasizing that the main achievement of the mechanism proposed in this paper is to induce the Higgs mechanism via a seesaw mechanism. If the seesaw mechanism is implemented in a two scalar doublets model, the standard Higgs mechanism is induced by the decoupling of the scales involved in the model. An intermediate energy scale in the $10^9\,\text{GeV}$ energy region is required. Depending on the details of the breaking of the unification group, the naturalness problem could be solved using the scale invariance argument presented in [23]. This issue is relevant to model building and will be considered in a forthcoming paper.

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